2.2 Laws of Logarithms

Question Paper

Course	CIE A Level Maths
Section	2. Logs & Exponentials
Topic	2.2 Laws of Logarithms
Difficulty	Hard

Time allowed: 60

Score: /45

Percentage: /100

Question 1a

(a) Evaluate

$$\log_2 8^2 + 3\log_2 16 - 2\log_2 2^5.$$

[2 marks]

Question 1b

(b) Evaluate

$$3\ln 2 + 2\ln 5 - \frac{1}{2}\ln 10\ 000,$$

giving your answer in the form $\ln p$.

[3 marks]

Question 2a

(a) Solve the equation

$$4^{3x+2} = 16^{x+6}.$$

[2 marks]

Question 2b

(b) Solve the equation

$$4^{2x+3} - 8 = 92$$

giving your answer to 3 significant figures.

[3 marks]

Question 3a

Solve the following equations, giving your answers in exact form.

(a) $4e^{3x-2} = 12$

[2 marks]

Question 3b

(b) $3e^{2x} + 8 = 14e^x$

[3 marks]

Question 4a

(a) Simplify

$$2\ln 3^4 + \ln 3^3 - \ln 9,$$

giving your answer in the form $a\ln b$, where a and b are integers to be found.

[2 marks]

Question 4b

(b) Write

$$2\log_a x + 3\log_a(x + 1) - \log_a 4(x + 2)$$

as a single logarithm.

[2 marks]

Question 5

(i) On the same axes, sketch the graphs of $y = e^x$ and $y = \ln x$.
   On each graph, label any points where the graph intersects the coordinate axes.
   Write down the equations of any asymptotes for each graph.

(ii) Write down the line of reflection between the graphs $y = e^x$ and $y = \ln x$.

[5 marks]

Question 6

Solve the equation

$$5^{2x} - 8 \times 5^x + 12 = 0,$$

giving your answers in the form $\log_a b$.

[3 marks]

Question 7

Solve the equation

$$6 \times 3^{x-1} = 6^{2x},$$

giving your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers to be found.

[5 marks]

Question 8

A ship sets sail from a harbour.

After some time, the ship's position is $(4\ln 3)$ km east of the harbour and $(3\ln 3)$ km north of the harbour.

Find the direct distance between the ship and the harbour at this time giving your answer in the form $(p\ln 3)$ km.

[4 marks]

Question 9

By writing 5 as $5\ln e$, show that

$$5\ln 2 + 5$$

can be written as $5\ln 2e$.

[3 marks]

Question 10

Solve the equation

$$\log_3(x + 4) = 4 + 2\log_3 x$$

giving your answers correct to 3 significant figures.

[3 marks]

Question 11

Solve the equation

$$2 \log_x(x + 2) = 3$$

giving your answer correct to 3 significant figures.

<div align="right">[3 marks]</div>